

Sequence and Series of a function

Sequences — A function whose domain is the set \mathbb{N} of natural numbers and Range a set of real numbers is called a real sequence.

Thus real sequence is denoted by symbolically as $S: \mathbb{N} \rightarrow \mathbb{R}$

Notation Since the domain for a sequence is always \mathbb{N} , a sequence is specified by the values S_n , $n \in \mathbb{N}$. Thus a sequence may be denoted as

$$\{S_n\}, n \in \mathbb{N} \text{ or } \{S_1, S_2, S_3, \dots, S_n, \dots\}$$

The values of S_1, S_2, S_3, \dots are called the first, second, \dots terms of the sequence.

The m th and n th terms S_m and S_n for $m \neq n$ are treated as distinct terms even if $S_m = S_n$

The number of terms in a sequence is always infinite.

In other words,

A sequence as an ordered set of real numbers can be put in a one-one correspondence with the set of natural numbers. However a sequence may have only a finite number of distinct elements.

For example

$$1. \{S_n\} = \{(-1)^n\}, n \in \mathbb{N}$$

Here $S_1 = -1$, $S_2 = 1$, $S_3 = -1$, $S_4 = 1$...
so that there are only two, $1, -1$ distinct elements.

$$2. \{S_n\} = \left\{ \frac{1}{n} \right\} n \in \mathbb{N}$$

Here $S_1 = 1$, $S_2 = \frac{1}{2}$, $S_3 = \frac{1}{3}$...

All elements are distinct.

The Range

The range or Range set is the set consisting of all distinct elements of a sequence, without repetition and without regard to the position of a term. Thus the range may be a finite or an infinite set, without ever being the null set.

Bounds of a sequence

Bounded above sequence

A sequence $\{S_n\}$ is said to be bounded above if there exists a real number K such that

$$S_n \leq K \quad \forall n \in \mathbb{N}$$

Bounded below sequences - A sequence $\{S_n\}$ is said to be bounded below if there exists a real number

k such that
$$s_n > k \quad \forall n \in \mathbb{N}$$

Bounded Sequence

A sequence is said to be bounded when it is bounded both above and below. K and k are respectively the upper and the lower bounds of the sequence.

Note A sequence is bounded iff its range is bounded

Also the bounds of the range are the bounds of the sequence.